LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

THIRD SEMESTER – **NOVEMBER 2013**

PH 3506 – MATHEMATICAL PHYSICS

Date : 06/11/2013 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

PART – A

Answer ALL questions: **(10 x 2 = 20 Marks)**

1. Find the square root of $-15-8i$.
2. Verify that $e^{-x}(\cos y - i \sin y)$ is analytic.
3. Find a unit normal vector of the cone of revolution $z^2 = 4(x^2+y^2)$ at the point (1, 0, 2).
4. State Green's theorem in the plane.
5. Check whether $\sin 2x \sin 3x$ is orthogonal in the interval $(-\pi, \pi)$.
6. What are odd and even functions?
7. Show that $\begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ -i/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$ is Hermitian.
8. Determine the rank of a matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{pmatrix}$
9. Using Lagrange's interpolation formula, find y(10) from the following data

x	5	6	9	11
y	12	13	14	16

10. Using Simpson's one third rule, evaluate $\int_0^1 y\,dx$ from the following data

x	0	0.25	0.50	0.75	1.00
y	1.0000	0.8000	0.6667	0.5714	0.5000

PART – B

Answer any FOUR questions: **(4 x 7.5 = 30 Marks)**

11. Show that (a) $\cosh z = \cosh x \cos y + i \sinh x \sin y$
    (b) $\sinh z = \sinh x \cos y + i \cosh x \sin y$

12. State and prove Stoke's theorem.

13. Find the Fourier series of the function $f(x) = \begin{cases} 0, & if -2 < x < -1 \\ k, & if -1 < x < 1 \\ 0, & if \quad 1 < x < 2 \end{cases}$

14. Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and find its inverse.

15. Fit a straight line by method of least squares for the following data

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

PART – C

Answer any FOUR questions: **(4 x 12.5 = 50 Marks)**

16. (a) State and prove Cauchy's integral theorem. (4.5)

(b) Using Cauchy's integral formula, evaluate $\oint \frac{z^2+1}{z^2-1} dz$ counterclockwise around the circle $|z-1| = 1$. (4)

(c) Evaluate $\oint \frac{z^4-3z^2+6}{(z+i)^3} dz$ counterclockwise around a unit circle with centre at the origin. (4)

17. (a) Prove that $\nabla . \nabla \times \mathbf{F} = 0$, where **F** is a three dimensional vector in Cartesian coordinates. (4)

(b) Verify Gauss-divergence theorem for the vector $\vec{A} = x^2\,\vec{i} + y^2\,\vec{j} + z^2\,\vec{k}$ taken over the cube $0 \le x, y, z \le 1$. (8.5)

18. (a) Find the Fourier cosine and sine integral of $f(x) = e^{-kx}$ where $x > 0, k > 0$. (11)

(b) Evaluate $\int_0^{\infty} \frac{\omega \sin 3\omega}{\omega^2+9} d\omega$ (1.5)

19. Find the eigen values and eigen vectors of $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$ (5+7.5)

20. Find the solution to four decimals, of the system

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

using Gauss-Seidel method.

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